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Optimal hazard reduction for recurrent episodes of pest or disease incursion and eradication

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Pest and disease incursions can impose significant costs of forgone production and trade. Additionally, there are costs of management activities such as hazard reduction, eradication and control. The assessment of these costs and the determination of the optimal management response have attracted attention in recent years. Minimising the expected present value of the total of the above mentioned costs is one of the criteria for determining how to protect the economy from the effects of unmitigated hazards. It involves a trade-off between additional expenditure on hazard reduction and the expected benefits from a reduced hazard rate of future incursions. In this paper, an expression is developed for the expected discounted present value of all future incursion and hazard reduction costs for the case of recurrent episodes of pest or disease incursion and eradication. The optimal expenditure on hazard reduction for a risk neutral decision maker is obtained by minimising this expression with respect to hazard reduction activity, subject to the relationship between hazard reduction activity and the hazard rate. For decision makers that are not risk neutral, the variance of the discounted present value of all future costs as a function of the hazard reduction activity is also derived. An example is provided to illustrate determination of the optimal hazard reduction activity.

Introduction

Pest and disease incursions have attracted increased attention in recent years. Economic analysis has been seen as essential to a full understanding of the problem of pest and disease incursions by providing a consistent and comprehensive assessment of the benefits and costs of - usually publicly funded - control alternatives (eg. Perrings *et al.* (2000 and 2002) and Evans (2003)).

In this paper an economic analysis of hazard reduction of pest or disease incursions characterised by recurrent episodes of incursion and eradication is put forward. Henceforth disease will be used to indicate either a pest or a disease. It is assumed that a disease can be economically eradicated in a known time and that episodes of incursion and eradication are the outcome of a stochastic incursion process with a constant hazard rate per time unit. It is further assumed that initially there is no disease and that each incursion results in an identical and constant flow of costs - including the eradication costs - till eradication is achieved. For a given hazard rate, there is an expected discounted present value of the cost of recurrent episodes of incursion and eradication over all time. This expected cost also depends on the discount rate, the time required for eradication and the level of the constant flow of incursion costs. Now assume that the hazard rate can be reduced by hazard reduction activity at an additional constant flow of hazard reduction costs and that the relationship between hazard rate and hazard reduction cost is known. The duration of the hazard reduction activity and the associated cost can be the time from eradication to incursion, i.e., a stochastic variable dependent on the hazard rate, or the whole infinite time horizon. The expected total cost of incursion and hazard reduction then depends on hazard reduction activity. In this paper we derive expressions for both the expected value and the variance of the discounted present value of hazard reduction and incursion costs. For a risk neutral manager, the optimal level of hazard reduction activity is where this expected value is smallest. For managers who are not risk neutral, the variance will also play a role in their decision-making as they may, for example, trade off an increase in expected cost for a decrease in the variance of cost.

For simplicity, in this paper only hazard reduction is considered, while all other management activities are assumed to be fixed and known. However, the method described in this paper can be extended to include surveillance activity that reduces the time to detect an incursion and thus the time needed to achieve eradication. Also, the method can accommodate increasing costs of incursions over the period till eradication as a result of spread of the disease. Both these features were taken into account in the determination of optimal surveillance activity for the Papaya Fruit Fly in Australia (Kompas *et al.*, 2003).

Related earlier work is Jensen (2002), in which optimal hazard reduction is analysed dynamically for the case of a single incursion that is never to be eradicated; and Shogren (2000), in which a static approach is used to analyse optimal hazard reduction. Dynamic analysis of additional infections from across borders for a pest or disease that is already endemic here and is never to be eradicated can be found in Beare and Hinde (2001) and Leung *et al.* (2002).

Method

Assume that initially (at time $t = 0$) there is no disease, but an incursion can occur at any time with a probability per time unit (the hazard rate), p , that is independent of time. The corresponding probability density of first incursion at time t is assumed to be exponential, pe^{-pt} . Hazard reduction is contemplated at a constant cost flow, h . The hazard rate is assumed to be a monotonically declining function of the cost of hazard reduction, $p = p(h)$. An incursion is assumed to result in a given constant flow of costs, c , that includes eradication costs, until eradication T time units later. An expression is then derived for the expected discounted present value of the sum of the costs of hazard reduction and incursion over an infinite time horizon as an explicit function of hazard reduction activity. For a risk neutral decision maker, the optimal level of hazard reduction is found when this expected cost is smallest. The variance of the discounted present value of all costs is also derived.

The continuous process of incursion at uncertain time and its subsequent eradication, followed by repeat incursion, eradication, *et cetera*, is shown in Figure 1.

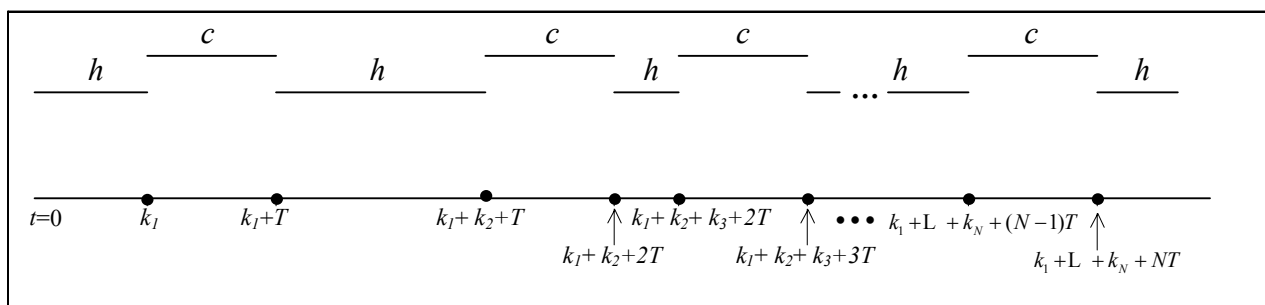


Figure 1. The recurring process of disease incursion and eradication with associated hazard reduction and incursion costs.

In Figure 1, the decision variable is the hazard reduction cost, h . This cost is in the first instance assumed to be positive and constant during all future periods from eradication to the next incursion, and zero at other times. Incursion costs, c , are positive and constant during all future periods from incursion to the next eradication, and zero at

other times. The arrival time of the i th incursion, measured from the last eradication, is indicated by k_i .

Now consider a particular realisation of incursion arrival times $\{k_i, i=1,2,\dots\}$. The discounted present value of the flow of costs of hazard reduction and incursion, for this series of arrival times, at a constant discount rate r , is:

$$C(k_1, k_2, L) = \underbrace{\int_0^{k_1} h e^{-rt} dt}_{\text{hazard reduction cost up to } k_1} + \underbrace{\int_{k_1}^{k_1+T} c e^{-rt} dt}_{\text{incursion cost till eradication at } k_1+T} + \underbrace{\int_{k_1+T}^{k_1+k_2+T} h e^{-rt} dt + \int_{k_1+k_2+T}^{k_1+k_2+2T} c e^{-rt} dt}_{\text{total cost of second episode}} + \underbrace{L}_{\text{cost of subsequent episodes}} \quad (1)$$

Define $G(t) = \int_0^t h e^{-r\tau} d\tau + \int_t^{t+T} c e^{-r\tau} d\tau$, as the discounted present value of the cost over the first incursion and eradication episode, if incursion occurs at t ; also define $a_i = \exp(-r((i-1)T + \sum_{l=1}^{i-1} k_l))$ for all $i > 1$, and $a_1 = 1$, as the discount factors applying to the cost over each episode. Expression (1) can now be written as

$$C(k_1, k_2, \Lambda) = \sum_{i=1}^{\infty} a_i G(k_i) \quad (2)$$

Denote $P(k_1, k_2, \Lambda)$ as the joint probability for the series $\{k_i, i=1,2,\dots\}$. This joint probability is the product of the individual probability of each k_i , because the time of incursion since the immediately preceding eradication is statistically independent of the history up to that time. The joint probability can thus be written as $P(k_1, k_2, \Lambda) = p e^{-pk_1} \cdot p e^{-pk_2} \cdot \Lambda \cdot p e^{-pk_3} \cdot \Lambda \dots$

The expected cost

The expected discounted present value of the total cost of hazard reduction and incursion, taking into account all possible values of the series $\{k_i, i=1,2,\dots\}$, is

$$\begin{aligned} E[C] &= \int_0^{\infty} \int_0^{\infty} L \int_0^{\infty} [P(k_1, k_2, L) C(k_1, k_2, L)] dk_1 dk_2 L \\ &= \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} L \int_0^{\infty} [P(k_1, k_2, L) a_i G(k_i)] dk_1 dk_2 L \\ &= \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \left[p e^{-pk_1} \cdot p e^{-pk_2} \cdot L \cdot p e^{-pk_i} \cdot e^{-r((i-1)T + \sum_{l=1}^{i-1} k_l)} G(k_i) \right] dk_1 dk_2 L dk_i \end{aligned}$$

$$\begin{aligned}
 &= \sum_{i=1}^{\infty} p^{i-1} e^{-(i-1)rT} \prod_{l=1}^{i-1} \left[\int_0^{\infty} e^{-(p+r)k_l} dk_l \right] \left[\int_0^{\infty} p e^{-pk_i} \cdot G(k_i) dk_i \right] \\
 &= \left[\int_0^{\infty} p e^{-pt} \cdot G(t) dt \right] \sum_{i=1}^{\infty} \left[p e^{-rT} / (r+p) \right]^{i-1} \\
 &= \left[\int_0^{\infty} p e^{-pt} G(t) dt \right] / \left[1 - p e^{-rT} / (r+p) \right]
 \end{aligned}$$

where $\int_0^{\infty} p e^{-pt} G(t) dt$ is the expected discounted present value of the total costs of the first episode of incursion and eradication.

Noting that $\int_0^{\infty} p e^{-pt} G(t) dt = [h/(r+p)] + [cp(1-e^{-rT})/((r+p)r)]$ and recalling that $p = p(h)$, it follows that

$$E(C) = \frac{h}{r} [1 - \alpha(p(h))] + \frac{c}{r} \alpha(p(h)), \quad (3)$$

where $\alpha(p(h)) = [1 - e^{-rT}/r] / [1/p(h) + (1 - e^{-rT})/r]$ can be interpreted as the expected “discounted” time from incursion to eradication as a proportion of the expected “discounted” time for a whole episode.

So, equation (3) gives the expected discounted present value of the sum of the hazard reduction and incursion costs. From this equation it is a straightforward exercise to find the level of hazard reduction activity for which this expected value is smallest (see the example on page 9). The first order condition for optimality can be readily obtained as:

$$\frac{\partial E}{\partial h} = \frac{1 - \alpha(p(h))}{r} + \frac{(c-h)}{r} \frac{\partial \alpha(p(h))}{\partial h} = 0.$$

This condition states that in the optimum, the increase in the present value of all costs from an increase in hazard reduction activity, given the proportion $\alpha(p(h))$, must be exactly offset by the decrease from the induced change in that proportion, and analogous conditions hold for a decrease in hazard reduction activity.

The expected cost in special cases

The analysis above is based on the assumption that hazard reduction would not be carried out over the time from incursion to eradication. If it is assumed that hazard reduction applies at all times, then the expected discounted present value of hazard reduction and incursion costs is:

$$E(C) = \frac{h}{r} + \frac{c}{r} \alpha(p(h)). \quad (4)$$

Note, if the eradication time T tends to infinity, equation (3) becomes

$$\lim_{T \rightarrow \infty} E(C) = [h + [p(h)/r] \cdot c] / [r + p(h)]. \quad (5)$$

This expression corresponds with the expression obtained by Jensen (2002) for the case of a single incursion that is never eradicated.

The variance of costs

The smallest expected total cost is the relevant criterion for a risk neutral decision maker. However, a risk-averse decision maker will be concerned also about the variance of total costs, and decide on the basis of his/her preference for combinations of the expected total cost and the variance of the total cost. For example, a higher expected cost might be traded off against a smaller variance of that cost.

The formula for the variance of the total costs can be derived as follows. By definition, the variance of C denoted by $Var(C)$ is equal to $E(C^2) - [E(C)]^2$. From equation (2),

$$C^2 = \left[\sum_{i=1}^{\infty} a_i G(k_i) \right]^2 = \sum_{i=1}^{\infty} a_i^2 [G(k_i)]^2 + 2 \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} a_i a_j G(k_i) G(k_j).$$

So, $E(C^2) = E\left(\sum_{i=1}^{\infty} a_i^2 [G(k_i)]^2 \right) + 2E\left(\sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} a_i a_j G(k_i) G(k_j) \right)$, where

$$\begin{aligned} E\left(\sum_{i=1}^{\infty} a_i^2 [G(k_i)]^2 \right) &= \int_0^{\infty} \int_0^{\infty} L \int_0^{\infty} L \left[P(k_1, k_2, L) \sum_{i=1}^{\infty} a_i^2 [G(k_i)]^2 \right] dk_1 dk_2 L \\ &= \sum_{i=1}^{\infty} \int_0^{\infty} L \int_0^{\infty} \left[p e^{-pk_1} \cdot L \cdot p e^{-pk_i} \cdot e^{-2r((i-1)T + \sum_{l=1}^{i-1} k_l)} [G(k_i)]^2 \right] dk_1 dk_2 L dk_i \\ &= \sum_{i=1}^{\infty} p^{i-1} e^{-2(i-1)rT} \prod_{l=1}^{i-1} \left[\int_0^{\infty} e^{-(p+2r)k_l} dk_l \right] \left[\int_0^{\infty} p e^{-pk_i} \cdot [G(k_i)]^2 dk_i \right] \\ &= \left[\int_0^{\infty} p e^{-pt} \cdot [G(t)]^2 dt \right] \sum_{i=1}^{\infty} \left[p e^{-2rT} / (2r + p) \right]^{i-1}; \end{aligned}$$

$$\begin{aligned}
 E\left(\sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} a_i a_j G(k_i) G(k_j)\right) &= \int_0^{\infty} \int_0^{\infty} L \int_0^{\infty} L \left[P(k_1, k_2, L) \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} a_i a_j G(k_i) G(k_j) \right] dk_1 dk_2 L \\
 &= \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \int_0^{\infty} L \int_0^{\infty} \left[p e^{-pk_1} \cdot L \cdot p e^{-pk_j} \cdot e^{-r[(i-1)T + \sum_{l=1}^{i-1} k_l]} \cdot e^{-r[(j-1)T + \sum_{l=1}^{j-1} k_l]} G(k_i) G(k_j) \right] dk_1 L dk_j \\
 &= \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \left\{ p^{j-2} e^{-(i+j-2)rT} \prod_{l=1}^{i-1} \left[\int_0^{\infty} e^{-(p+2r)k_l} dk_l \right] \prod_{l=i+1}^{j-1} \left[\int_0^{\infty} e^{-(p+r)k_l} dk_l \right] \right. \\
 &\quad \left. \times \int_0^{\infty} \int_0^{\infty} p e^{-(p+r)k_i} p e^{-pk_j} G(k_i) G(k_j) dk_i dk_j \right\} \\
 &= \left[\int_0^{\infty} p e^{-(p+r)t} G(t) dt \right] \left[\int_0^{\infty} p e^{-pt} G(t) dt \right] e^{-rT} \sum_{i=1}^{\infty} \left\{ \left[\frac{(r+p)e^{-rT}}{2r+p} \right]^{i-1} \sum_{j=i+1}^{\infty} \left[\frac{p e^{-rT}}{r+p} \right]^{j-2} \right\}.
 \end{aligned}$$

Now noting that $\int_0^{\infty} p e^{-pt} \cdot [G(t)]^2 dt = \frac{p}{r^2} \left[\frac{h^2}{p} - \frac{2h[h-c(1-e^{-rT})]}{r+p} + \frac{[h-c(1-e^{-rT})]^2}{2r+p} \right]$,

and $\int_0^{\infty} p e^{-(p+r)t} G(t) dt = \frac{p}{r} \left[\frac{h}{r+p} - \frac{h-c(1-e^{-rT})}{2r+p} \right]$, it follows that

$$\begin{aligned}
 E(C^2) &= \frac{p}{r^2} \left[\frac{h^2}{p} - \frac{2h[h-c(1-e^{-rT})]}{r+p} + \frac{[h-c(1-e^{-rT})]^2}{2r+p} \right] \Bigg/ \left[1 - \frac{p e^{-2rT}}{2r+p} \right] \\
 &+ \frac{2}{r^2} \left[\frac{h}{r+p} - \frac{h-c(1-e^{-rT})}{2r+p} \right] \left[\frac{h}{p} - \frac{h-c(1-e^{-rT})}{r+p} \right] p^2 e^{-rT} \Bigg/ \left\{ \left[1 - \frac{p e^{-rT}}{r+p} \right] \left[1 - \frac{p e^{-2rT}}{2r+p} \right] \right\}, \quad (6)
 \end{aligned}$$

where $p = p(h)$. Having the equations (3) and (6), the variance $Var(C) = E(C^2) - [E(C)]^2$ can be obtained.

Notes:

(1) If $h = c$, then the cost flow is constant all the time (see Figure 1), no matter whether a disease incursion occurs and no matter when incursions occur. So, there is no variability in total cost, that is, the variance, $Var(C) = 0$ in this case. This can also be easily proved analytically.

(2) If the eradication time T tends to infinity (i.e., the disease can never be eradicated), equation (6) becomes

$$\lim_{T \rightarrow \infty} E(C^2) = \frac{p}{r^2} \left[\frac{h^2}{p} - \frac{2h(h-c)}{r+p} + \frac{(h-c)^2}{2r+p} \right], \text{ thus,}$$

$$\begin{aligned} \lim_{T \rightarrow \infty} Var(C) &= \lim_{T \rightarrow \infty} E(C^2) - [\lim_{T \rightarrow \infty} E(C)]^2 \\ &= \frac{p}{r^2} \left[\frac{h^2}{p} - \frac{2h(h-c)}{r+p} + \frac{(h-c)^2}{2r+p} \right] - \left[\frac{h + [p(h)/r] \cdot c}{r + p(h)} \right]^2 = \frac{p(h-c)^2}{(2r+p)(r+p)^2}. \end{aligned} \quad (7)$$

Example

A hypothetical example is used to illustrate how the optimal level of hazard reduction can be obtained using the formulae derived above. The formulae require the values of parameters of the discount rate r , the eradication cost c , and the time from incursion to eradication T , and the functional relationship between the hazard rate p and the hazard reduction activity h .

Let $r = 0.05$ per year, $T = 1$ year, $c = 2$ million dollars per year, and the monotonically decreasing relationship between p and h given by $p(h) = 0.4e^{-0.08\sqrt{h}}$. The function $p(h)$ is depicted in Figure 2.

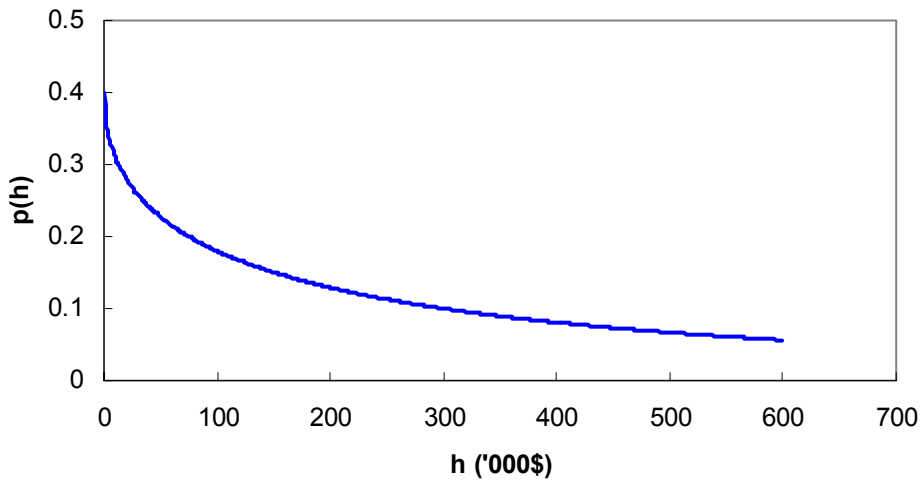


Figure 2. The functional relationship between the hazard rate p and the level of the hazard reduction activity h .

The results for the optimal level of hazard reduction are shown in Table 1 for four cases, i.e., $T = 1$; $T \rightarrow \infty$; h applying all the time; and h applying only in disease-free periods.

Optimal hazard reduction activity when h applies all the time is lower than that when h applies in disease free periods only. The percentage of the reduction of the optimal h when h applies all the time compared with when h applies in disease free periods only depends on the value of T , i.e., the time from incursion to eradication. The larger T , the larger is the percentage reduction in the optimal h . When $T \rightarrow \infty$, the optimal h is

\$329,675 when h applies all the time, compared with the optimal h of \$683,962 when h applies in disease free periods only, that is a 52% reduction of the optimal h . The value of the optimal h also depends on the value of T and the eradication cost c . Shown in Figures 3 and 4 are the results of the relationships between the optimal h and the parameter T and between the optimal h and the parameter c , respectively. Either higher T or higher c brings higher total cost during the incursion periods; in these cases, a rational decision maker should increase spending on the hazard reduction activity to reduce the frequency of disease incursion periods. This is the reason why higher T leads to higher optimal h when all other parameters are kept constant; and why higher c leads to higher optimal h , as shown in Figures 3 and 4.

Table 1. Optimal hazard reduction activity and the optimal expected cost

	Optimal h (\$ per year)*	Optimal p (yearly)**	Optimal $E(C)$ (\$)
$T = 1$			
h applies in disease free periods only	114,770	0.17	7,651,954
h applies all the time	101,960	0.18	7,966,193
$T \rightarrow \infty$			
h applies in disease free periods only	683,962	0.05	26,755,580
h applies all the time	329,675	0.09	32,664,810

* This compares with the incursion and eradication cost $c = 2,000,000$ dollars per year.

** This compares with $p = 0.4$ when $h = 0$ (i.e., the unmitigated hazard scenario).

Figure 5 shows the expected total cost $E(C)$ and the variance of total cost $Var(C)$ against hazard reduction activity h , where $E(C)$ is at its minimum around $h_0 = \$115,000$. Any h higher than h_0 leads to higher $E(C)$ but lower $Var(C)$. When h increases to c , $Var(C)$ becomes zero but $E(C)$ becomes much higher. When the decision maker is not risk neutral, it is his/her preferences over the combinations of expected cost and variance that determine what activity level will be optimal, either with a higher $E(C)$ but lower $Var(C)$ or a lower $E(C)$ but higher $Var(C)$.

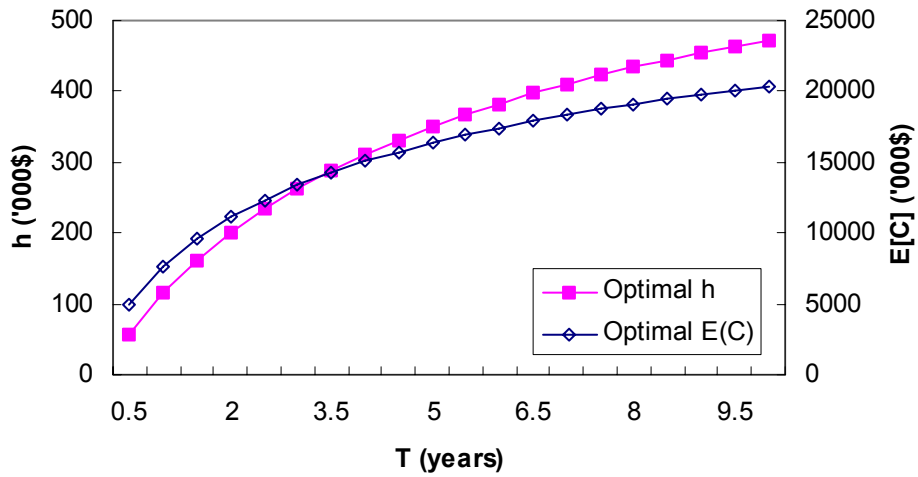


Figure 3. The relationship between optimal hazard reduction activity h and the eradication time parameter T , and the relationship between the optimal $E(C)$ and the parameter T .

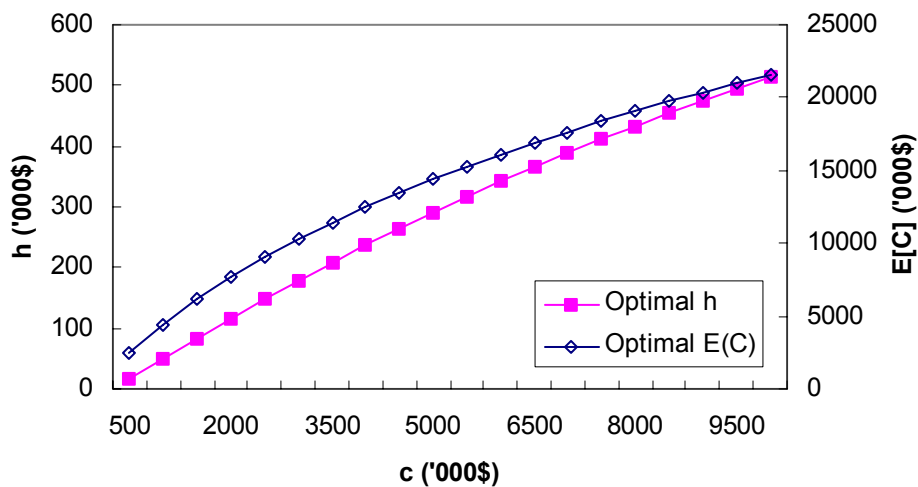


Figure 4. The relationship between the optimal hazard reduction activity h and the incursion and eradication cost parameter c , and the relationship between the optimal $E(C)$ and the parameter c .

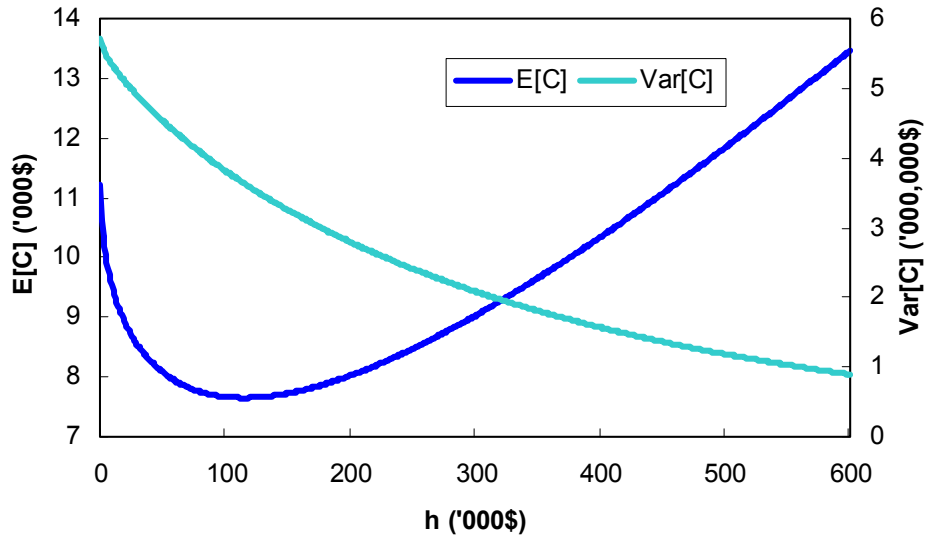


Figure 5. The relationship between the expected cost $E(C)$ and the hazard reduction activity h , and the relationship between the variance $Var(C)$ and the activity h .

Discussion

To determine the optimal level of hazard reduction activity, it is required to know the incursion and eradication cost c ; the time from incursion to eradication T ; the functional relationship $p(h)$ between the measure of hazard reduction and the hazard rate; the time periods during which hazard reduction costs are incurred; and the discount rate.

The cost c does not have to be constant over the periods from incursion to eradication. In other words, c can be a function of time t , that is, $c = c(t)$. For example, $c(t)$ can be monotonically increasing, and can be linked to the biological dynamics of disease spread (see Kompas *et al.*, 2003). For the formulae presented in this paper to continue to hold, the function $c(t)$ needs to be the same in all incursion episodes.

In general, there could be a relationship between the eradication time T and the incursion and eradication cost c , that is, $T = T(c)$. Higher c may result in a shorter eradication period T , because it is reasonable to assume that the higher the eradication activity, the earlier the disease is eradicated. This aspect is not discussed in this paper, but has been taken into account in our other work (Cao and Klijn, 2004).

In this paper it is assumed that there is no uncertainty about the cost c and eradication time T . In reality, c and T may not be known beforehand, and can be stochastic. For stochastic c , all formulae obtained in this paper still hold with just replacing the value of c by the expected value of c (it is reasonable to assume that the decision maker knows

the expected value of c). For stochastic T , however, the analysis is more complicated and is currently being investigated.

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